French Bilateral Trade Model and Factor Analysis on Japan's Bilateral Trades by *O.L.S.* Method and Economic Policies

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Contents

1. Introduction

In 1980s, the trade frictions between the Japan and other countries, especially both the United States and the Europe, have increased extremely owing to the large disequilibriums among these countries.(1) The caused of aggravation of disequilibrium are considered as the following three factors: cyclical factors, policy factors and structural factors.(2) However, we are not going to analyse these three factors(3) in detail in this article, we intend to take consideration of only one aspect on the causes of aggravation of disequilibrium, that is, the price discrimination and the products differenciation in the international trades between the Japan and both the United States and the France. Actually, this would be the problems of almost all Japanese exports of manufactured goods to the United states and to the France. Therefore, in Section 2 of this article we should like to examine profoundly a new analytical frameworks(4) by Jean-Marc Siroen, Professor of the University of Paris IX Dauphine: "Discrimination of Prices, Differenciation of Products and International Trade" in the Revue Economique, N° 3, May, 1986. The new theoretical analysis of bilateratel trades based on the monopolistic competition is very significant and useful for the explanation of Japan's actual export and import between both the United States and the France in spite of the complete(5) neglect of empirical studies founded on the statistical data. In order to carry out actual empirical analysis on Japan's import and export towards the United States and the France, we are going to execute a factor analysis on Japan's bilateral trades between both the United States and the France. Therefore, in Section 3 of this article, we will intend to explain a theoretical frameworks[6] of factor analysis in detail and in Section 4, we are going to apply the real statistical data[7] on Japan's import and export towards both the United States and the France[8] during the period of the fixed exchange system from 1956I to 1971II and that of the flexible exchange system from 1971III to 1986IV[9] on the basis of factor analysis by the least squares method.

Note

(1) Economic Planning Agency, Japanese Government, Economic Survey of Japan, 1988, pp. 63–67.
(2) Economic Planning Agency, op. cit., p. 68.
(3) Economic Planning Agency, op. cit., pp. 68–87.
(4) Jean-Marc Siroen, ≪Discrimination des prix, différenciation des produits et échange international≫, Revue Economique,—N° 3, mai 1986, pp. 489–520.
(5) J.M. Siroen, op. cit., pp. 517–518.
(6) Kazuo Nakatani, Tahenryo-Kaiseki (Multi-variate Analysis, Shinyo-Sha, Tokyo, 1978, pp. 1–57.
(7) International Monetary Fund, International Financial Statistics, January 1952–April 1988. Bank of Japan, The Bulletin of Economic Statistic, January 1953–April 1988.
(8) JMA Research Institute Inc. JRI Personal Computer Soft-Ware Library, Factor Analysis and Principal Component Analysis System. V2.0 1987. 6066. MS/DOS.
(9) Sadao Suwa, The Exchange-Control Model in France and Estimations of Japan's import functions of CES type from the U.S. and the France and Economic Policies, Waseda Economic Papers, Graduate School of Economics, Waseda University, 1987, pp. 1–28.

2.

The approach[1] that will be proposed is based on the extention of the prices-discrimination model to the problems which had not been raised in the initial presentations (in particular, that of Joan Robinson[2]).

The discrimination is possible and favourable for the profit of firm while:

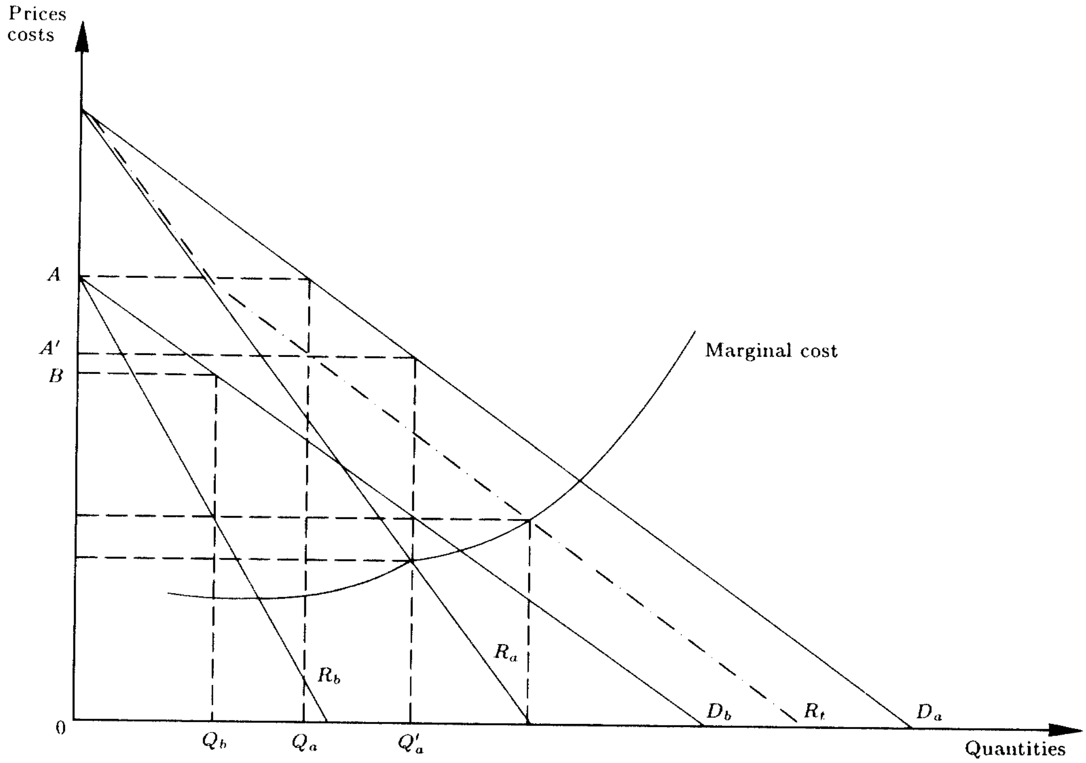
—for a certain product, the firm identifies many independent demand curves, and therefore the number of markets;

—the resales from a market to the other are impossible or too expensive (relatively to the price differentials).

The diagram 1 shows the results of discrimination: the firm carries out the highest prices on the market where the demand is the most inelastic (market a); by satisfying many markets.

The firm increases its marginal revenue, therefore its marginal cost (which is equal to the former), and then, the volume of production (in the hypothesis maintained here of increasing marginal cost). Finally, it ameliorates the rate of return(3).

Graph 1



D_a, D_b, R_a and R_b

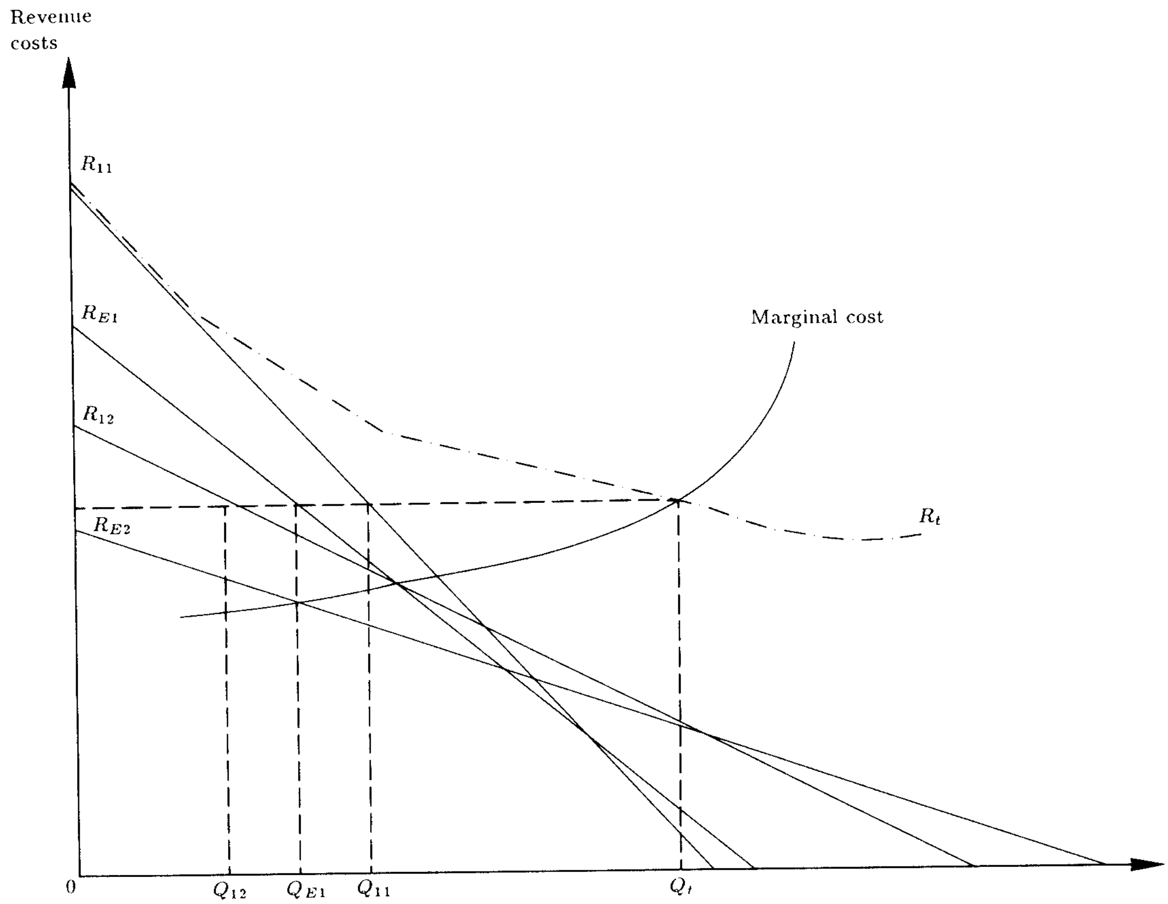
: demand and marginal revenue curves on the two markets a and b.

R_t : total marginal revenue curve.

Optimization rule: equalization between marginal revenue and marginal cost.

If the firm sells only in the markets a, the realized price is $0A'$ and the sold quantities are $0Q'_a$.

Graph 2



R_{11}, R_{12} : marginal revenue curves of the products 1 and 2 in the domestic market
R_{E1}, R_{E2} : marginal revenue curves of the products 1 and 2 to the export
R_t : total marginal revenue curve.

If the firm sells in the two markets, the prices are $0A$ and $0B$ in the markets a and b. The sold quantities are $0Q_a$ and $0Q_b$ in the market a and b. The total production is $(0Q_a + 0Q_b)$.

It will be supposed that the price of a certain product could only be discriminated by the export.

Most assuredly, certain cost can be specific for the market (transport costs, ...). They would be imputed to the price; then, represented demand curves are therefore net of export-costs.

One supposes now that, from a certain technic which is related to some marginal cost curve, it is possible to produce many different products not to be substitutable by consumer (independent demands).

The utillized schema to represent the discrimination can be therefore retaken, because all the conditions of discrimination are reunited: independent demands, impossible resales. Of course, the different products can tolerate specific costs, when the marginal cost curve only represents the common costs. However the schema would not be retaken if these costs specific to the products are imputed to the price as if they were the costs specific to the market. The demand curves are net of specific costs.

Then, one represents graphically (graph 2) the situation where the firm considers to sell in two markets (domestic market and export market) two products made of the same technique of production.

Program of Optimal Production

Product / Destination	1	2
Domestic market	$0Q_{11}$	$0Q_{12}$
Export	$0Q_{E1}$	0

Total production: $0Q_t = 0Q_{11} + 0Q_{12} + 0Q_{E1}$ (For graphical clearness, all demand curves have not been traced.)

$0Q_{11}$ of product 1 is sold in place and $0Q_{E1}$ of the same product is exported. The product 2 is simply sold in place for $0Q_{12}$. $0P$ are, therefore the prices relative to these different couples market-products. $0P$ would be:

$$0P_{11} > 0P_{E1} > 0P_{12}.$$

This extention of the model of discrimination can, then, explain the mode of simultaneous determination:

—programs of production (quantities of each product):

—price policies

—geographical destination of production.

It appears that the diversification of products utilizing the same factors of production with the same intensity, just like the market diversification, are capable to ameliorate the profitability of the firm.

These policies are, therefore, utilized by the firms which wish to maximize their profit.

We are going to show that the above analysed model is only partly modified when the proposed product belong to the same ≪group≫ and are only differenciated. According to Lancaster,[4] a product is defined by the characteristics that it incorporates. The product j can be therefore defined by the vector $e_j = (e_1, e_2, \ldots, e_n)$ where the e_n represent the quantities of characteristics n incorporated in the product j.[5] The products of one group have the same characteristics, but the different ≪varieties≫ or ≪presentations≫ incorporate them in different proportions. The characteristics are supposed quantifiable.

When the firm increases the quantity of a characteristic, one can expect to increase simultaneously the production cost and the quantity of demand. One can reasonably assume that the marginal efficiency of the quantity of incorporated characteristic n is decreasing (less increase of demand, when one incorporates more and more of n) at least from a certain value of e_m.

One can then pose:

$$p = p(q, e)$$

where p presents the selling price of the product, q the selling quantities and e the vector which defines the differenciation.

Suppose $c = c(q, e)$ where c represents the unitary costs of production, function of produced quantities and the quantities of incorporated characteristics.

The global profit is, then, defined by:

$$\pi = [p(q, e) - c(q, e)]q.$$

It can be maximized when (conditions of the first order):

$$(1) \quad \frac{\partial \pi}{\partial q} = q\frac{\partial p}{\partial q} + p - q\frac{\partial c}{\partial q} - c = 0$$

$$(2) \quad \frac{\partial \pi}{\partial e_n} = q\frac{\partial p}{\partial e_n} - q\frac{\partial c}{\partial e_n} = 0$$

The equation (1) is related to the traditional rule of equalization between marginal revenue and marginal cost. The equation (2) establishes the rule according to which one can increase his profits by incorporating more of characteristics n as long as the increases consecutive to this revenue can cover those of costs.

In this way, after Lancaster, it is the nature of incorporated characteristics that defines the product (≪group≫) and not the quantity. One has, therefore, $e_n \in [0, e_{\max}]$ where $e_{\max}$ represents the maximum quantity of characteristic n that it is technically and commercially possible to incorporate in product.

Or, on one hand, some secondary characteristics are able to be incorporated without being obligationary, that is, without defining the product and, on the other hand, a characteristic can define a product only if a minimum quantity of characteristic was incorporated: the size of an automobile or its power cannot decrease below a certain minimum. Besides, some characteristics are present—in almost identical quantity—in all the varieties of product and therefore do not participate to the differenciation.

One can have thus, for example:

$$e_n \in [e_{\min}, \ e_{\max}]$$

with $e_{\min}$: minimum quantity of incorporated characteristic.

One poses $e_{\min} = \bar{e}_n$.

The product is then defined by the vector $\bar{e}_j = (..., \bar{e}_n, \ ...)$, where $\bar{e}$ represent minimum quantities of characteristics to incorporate for obtaining a product of the group j.

The vector $\bar{e}_j$ is known and constitute a standard of reference. The differenciation is based on two elements: the supplementary quantity of ≪obligational≫ characteristics and the quantity of incorporated secondary characteristics.

A new vector is defined:

$$\hat{e} = (..., \ e_n - \bar{e}_n, \ ..., \ e_a, \ ...)$$

where

$$\bar{e}_n \leq e_n \leq e_{\max}$$

and $e_a \in [0, \ e_{a \ \max}]$ measures the quantity of incorporated secondary characteristic (being defined by the product).

Then, two types of costs can be defined:

—a cost of production relative to a vector known and common to all varieties: $\bar{e}$;

—a cost of differenciation relative to the vector $\hat{\boldsymbol{e}}$ of which the elements are specific to the variety.

There is not any relation between the price and the unitary cost of production.
Assume,

$\bar{c} = \bar{c}(q)$ the unitary cost of production and
$\hat{c} = \hat{c}(\hat{e})$ the unitary cost of differenciation.

The profit is written, then:

$$\pi = [p(q, \hat{e}) - \hat{c}(\hat{e}) - \bar{c}(q)] \cdot q$$

In order to maximize the profit, we must have (conditions of the first order):

$$(1') \quad \frac{\partial \pi}{\partial q} = q \cdot \frac{\partial p}{\partial q} + p - q \cdot \frac{\partial \bar{c}}{\partial q} - \bar{c}(q) = 0$$

$$(2') \quad \frac{\partial \pi}{\partial \hat{e}} = q \cdot \frac{\partial p}{\partial \hat{e}} - q \cdot \frac{\partial \hat{c}}{\partial \hat{e}} = 0$$

The marginal cost is, therefore, this time related to all varieties of product j, whatever their differenciation should be.

We can, then, define a revenue net of the costs of differenciation: $p' = p(q, \hat{e}) - \hat{c}(\hat{e})$.

The firm are going to increase the quantity of characteristic up to the point that the increase in net revenue would be null.

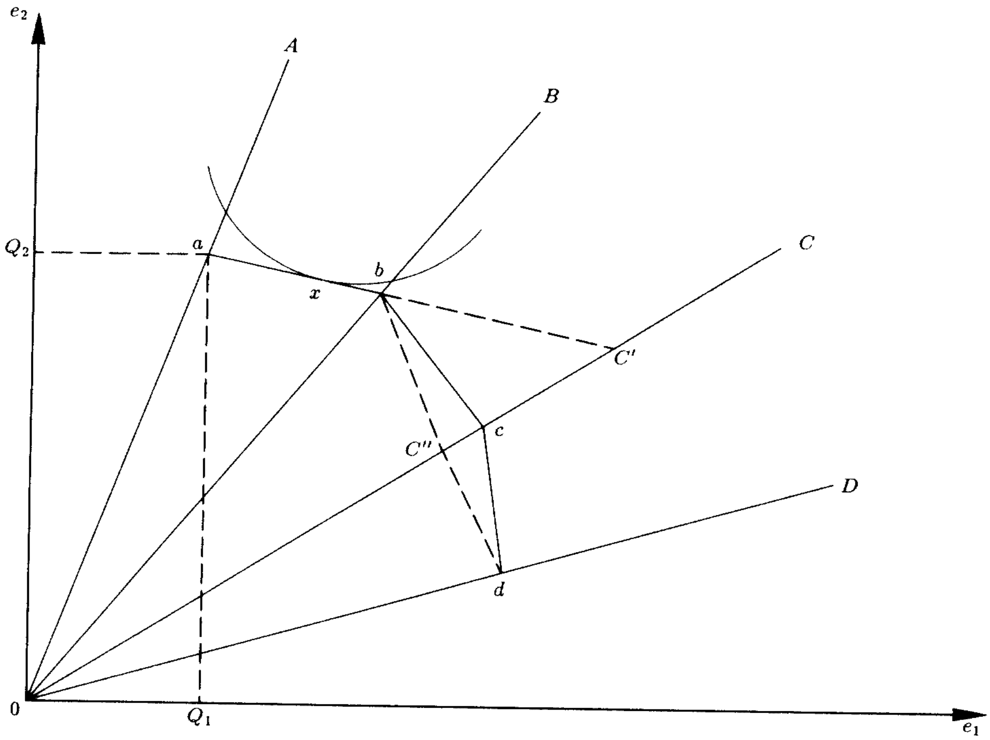
The assimilation between ≪selling costs≫ and ≪costs of differenciation≫ is possible if:

$$\frac{\partial p}{\partial \hat{e}} > 0 \Longrightarrow \frac{\partial p}{\partial \hat{c}} > 0$$

at least until to the limit $e_{\max}$.[6]

After the representation proposed by Lancaster[7] and the concept of ≪neighbourhood≫, the axes of the graph 3 represent the quantities of two characteristics. Four varieties are effectively produced: A, B, C and D. The points a, b, c and d represent the quantities of products which are able to obtain with a certain budgetary ressource (for example: 100 francs). Thus, with 100 francs, a consumer can buy $0a$ of A and dispose of $0Q_1$ characteristics e_1 and $0Q_2$ characteristics e_2. The consumer can equally buy a combination of goods in the condition, however, that the representative point of choice would be on the ≪market opportunities frontier≫: MOF. The real choice would be realized at the point of tangency of indifference curve of consumer with offered characteristics and of the ≪frontier≫, MOF (x) assumed perfectly known by all the consumers.

Graph 3



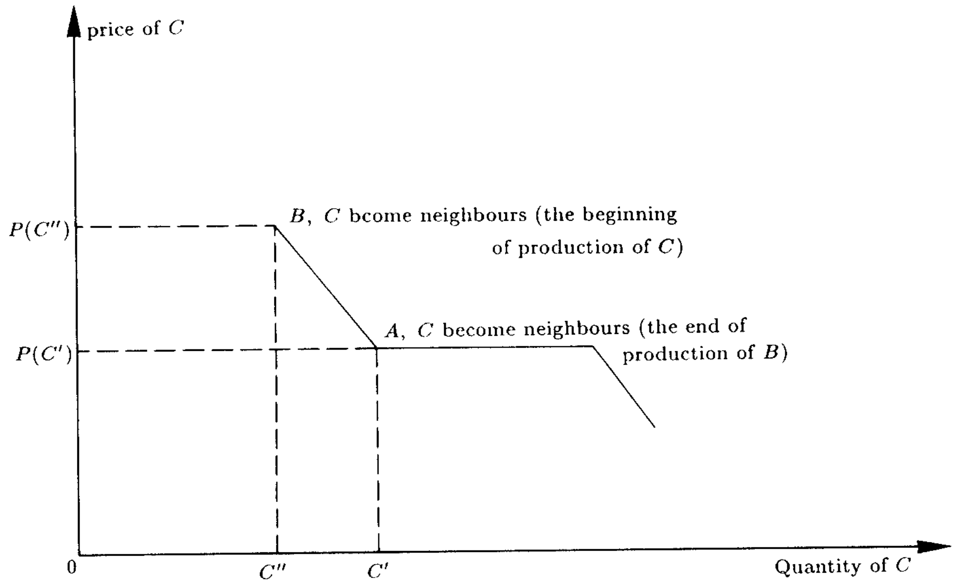
The products A, B, or B, C or C, D are called ≪neighbour≫, because they are, between them, in nearer competition than with the other varieties. A and D seem to be little substitutable in this example.

Geometrically, two varieties would be called ≪neighbour≫, if the straight segment which unites the representative points of varieties is mingled with the MOF curve.

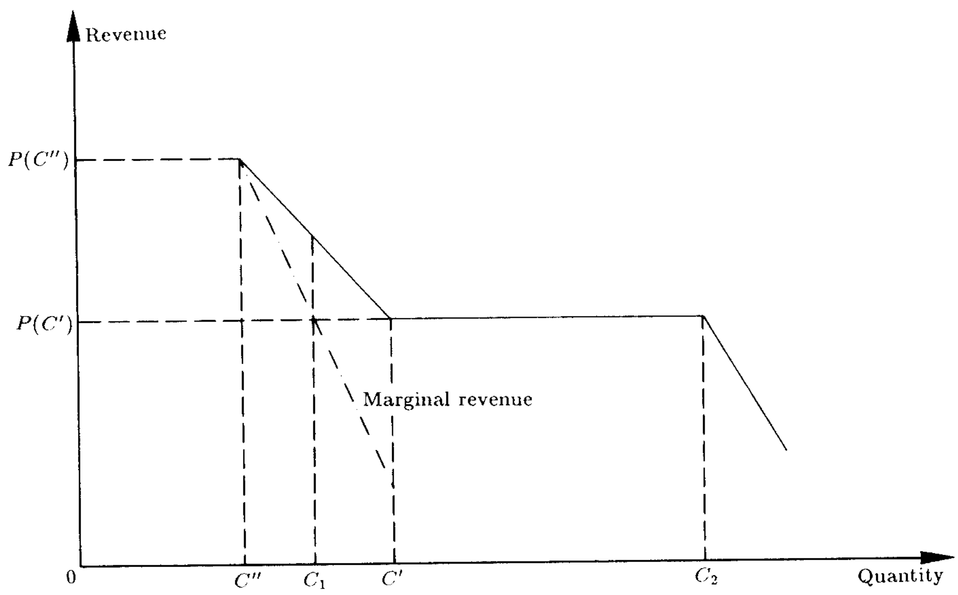
Assume that the price of the variety C would diminish without causing the fall of other prices for the firm producing C. (traditional hypothesis of Zero Conjectural Variations—ZCV).(8) The point would shift upward because, with 100 francs, the consumer can buy more of variety C.

Up to the point c' corresponding to a certain price $p(c')$ of c, B and C remain neighbours. However, beyond c', A and C would become

Graph 4



Graph 5



neighbours, because the segment that combines a with c' passes above b: all consumers of which the point of tangency of their indifference curve with the new curve of MOF exists between A and C must abandond the variety B of which the production would be, then, stopped. Inversely, B and C become neighbours only from c'': for a price superior to $p(c')$, B and D are neighbours and C is eliminated from the market.

There is, therefore, a certain interval of price — in consequence, of quantities — where some varieties are neighbour among them (graph 4). The demands for these near varieties are, then, intimately dependent: but they are relatively independent on the other varieties. The horizontal line on the graph, at the point of ordinate $p(c')$, illustrates the fact that, the production of B being abandanded, its market is partly captured by C. (the other part being appropriated by A).

In the graph 5, by the fact of the discontinuity of the demand curve, the marginal revenue curve is also discontinued, then it would be constant between 0_{C1} and 0_{C2}.

We have, therefore, a certain quantity 0_{C1} of C such as (graph 5):

$$R_m(0_{ci}) < R_m(0_{cj})$$

(with R_m representing the marginal revenue relative to a certain quantity).
where $0_{ci} \in [0_{C1}; 0_{C'}]$ and $0_{cj} \in [0_{C'}; 0_{C2}]$.
Then, it is possible that:

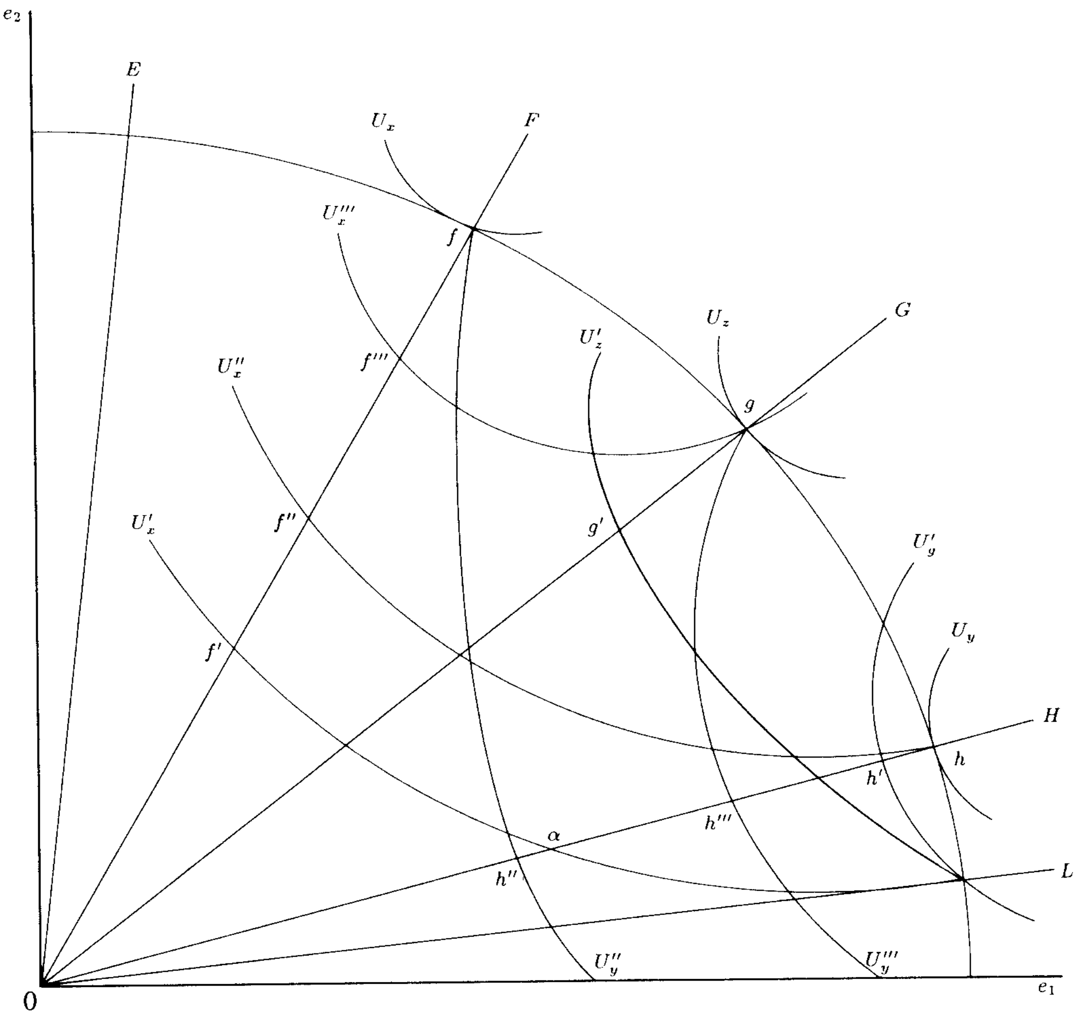
$$R(0_{ci}) = R_m(0_{a''})$$

while $R_m(0_{cj}) < R_m(0_{a''})$
where $R_m(0_{a''})$ shows the marginal revenue relative to the minimum quantity of variety A.

The firm can compare, then, two situations:

1. that where the firm starts the production of A when it has already produced 0_{ci} of C. In proportion to the increase in the production A, its marginal revenue would decrease and the optimization process required a supplementary production of C. But, it is possible that the equilibrium would be attained before the marginal revenue of C would reach the limit of discontinuity $p(c')$;
2. that where the firm produce 0_{C2} of C before the beginning of production A.

Graph 6



Besides, a minimum production $0''_a$ of A is demanded. When the firm begins the production A, a new discontinuity would appear at the level of marginal cost curve.

It is, in effect, possible that one could have:

$R_m(0_{ci})$ (or $R_m(0_{C2})$) $< C_m(0C_i)$ (or $C_m(0_{C2})$) and $R_m(0_{ci})$ (or $R_m(0_{C2})$) $< R_m(0_{a''})$

while: $C_m(0_{ci}$ (or 0_{C2}) $+ 0_{a''}) < R_m(0_{a''})$.

In this case, the firm will not produce A and will continue the production of C up to the point of equalization between the marginal revenue and the marginal cost.

If the firm produces A, the process can continue itself and the firm can consider to produce B, D, ... until the marginal revenue relative to the last produced variety would be equal to the marginal cost of total production.

In the graph 6, we can precisely illustrate the enlarged model of discrimination with many neighbour varieties produced by the same firm.(9) Substitute for the ≪frontier≫ initial MOF—related with monetary resources of consumer—≪a product differenciation curve≫: PDC, place of combinations of characteristics which can be produced at the effective cost.

In reality, it is the question of production possibilities curves.(10) Assume that the firm monopolizes the production of varieties placed between E and L (graph 6). The competition with other firms works only in the left part of E and in the right part of L. By simplification, it is supposed that it works only in the right part of L. Let U_x be the indifference curve with consumer's characteristics X. In competition, X consumes 0_f goods of F produced by the firm. However, in situation of monopoly, the firm is able to increase its price up to a certain limit without losing X because he maintains his buying of F (but decreases his buying quantities), or because he will go towards a neighbourhood variety, G for example, equally produced by the firm.

Suppose, in the first time, that G is not produced. If the indifference curves are homothetic, the price of F can rise up to the point where $0f'$ produced F are sold: if the price increases further, the consumer can obtain more of satisfaction in consuming a variety placed in the right of L. The monopoly have an effect to decrease the utility of X. Likewise, the eventual consumer Y of the variety H sees his utility pass from the curve V_y to the curve V_y'. One remarks that the price of F increases more relatively at the price of competition than the price of H, because H is nearer of the competitive substitutable products.

However, while F and H are produced effectively, they would go in competition for the consumer who directs toward H before buying outside of the firm.(11) If the price of H is the level indicated by $0h$ (—$p(0h)$—), the consumer X would increase his satisfaction in consuming $0h$ of H rather than $0f'$ of F. If the firm wishes to produce F, it would decrease the price of F to sell at least $0f''$ to X. It can increase the price of H and fix it at least at $p(0\alpha)$. It can take two reactions together. It is still possible that the firm abondons H, or that it proposes a new variety more distant from F, but nearer the competitive zone. Finally, the firm would give up the consumer Y.

When the firm produces only the variety F, the optimum price and, therefore, the optimum quantity are determined by the function of the rule of equalization between marginal revenue (net) and marginal cost. The optimization must be, however, realized within the interior to an interval of price and quantity (represented by $0f'$—$0f$ in neglecting the competition in the left of L). H would be produced as soon as a sufficiently low price could be in practice — in order to avoid the exterior substitutes —, but also enough high for it to be superior or equal to the marginal revenue of F, considering the eventual movements of F toward H. In function of the elasticities of their demands, the productions of F and H could increase jointly — and their prices could fall — up to the point of equalization between their marginal revenue and marginal cost determined in the other place. F being the longest way from the exterior competition, the firm could obtain relatively more important monopoly gains for this variety. The net price of F is higher. Eventually, other varieties could be in the same way produced.

We can find out once again, therefore, the principles of the process of optimization of initial discrimination model. The difference relates with the fact that the couple of price-quantity on which the firm can act is contained within the interior of certain limits linked up themselves with the neighbourhood of other varieties proposed by the firm. The initial model concerns not only with the choice of varieties, but also with that of markets.

The price discrimination can put again the interdependence among varieties of firm in question. The firm can, in fact, maintain the price of F at $P(0f')$ and the the price of H at $P(0h')$ in the same time — and, therefore, increase without concessions its monopoly gains — if the consumers place themselves in the different and separated markets by partitions. It would be enough not to propose H on the market of X, or to sell it at the price superior to $P(0\alpha)$. The market discrimination by export, therefore, enables the monopoly gains increase, and favour the increase in the number of varieties proposed by the firm. The monopoly gains relative to the ≪discutable≫ variety H increase themselves, which are equivalent to the increase in the interest to produce it.[12]

There are exchanges within the branches or within products when the produced varieties in two different countries are exchanged. The cited models would take in consideration that the number of firms and, therefore, the number of varieties had been limited by the economies of scale. As between exchanging nations, the conditions of production are near and the similar demands, the firms, then, the productions of varieties are assumed harmoneously distributed among these very near, so similar nations.(13) The models which belong to this approach(14) show that it is, then, possible to determine the volume and the price of trades. However, their ≪direction≫ remains still indetermined: we do not know which country must produce and export what variety: we know only that a variety can be produced only by a nation. The affectation of productions between the countries is, therefore, accidental as if, for the sames reasons, that of varieties to the firms is equally so. In discomposing two types of costs — costs of production and costs of differenciation (and of export) — inequally sensible to the economies of scale, we have seen that the number of firms would be as much as less relatively to the number of varieties than the economies of scale would have been important for the costs of production and unimportant for the costs of differenciation.(15)

It is, therefore, the problem to understand the reason why such and such variety would be produced by such a firm, then by such a nation.

We have seen that the differentials of costs of production would act little on the choice of varieties. We know that very differenciable products are made from very near techniques of production and put up with, in equal scale, the costs of similar production. They are, then, produced by the nations which seem to dispose of near factorial endowments.

The affectation of varieties to firms —, then to nations — depends, therefore, on the unequal capacity to incorporate certain characteristics: if the gross demand curves are able to be assumed identical for all firms, the net demand curves are different from the fact, as we have seen it, of the differenciation costs and of different export costs and of monopolies of incorporation of certain characteristics.

The comparative advantages in the incorporation of certain characteristics are defined in interior to the nation, but they would explain a very delicate definition of ≪aptitudes≫ or endowments of productive factors: the abandance of commercial frameworks, the network of services after selling, the investments in research and development, the flexibility of equipments are, for example, the elements which can explain the nature of them. The production of certain attributes could be, however, delocalized in the nations which dispose of a comparative advantages in their production.

In the following analysis, it is assumed that two varieties are only produced initially: F in the nation 1 and H in the nation 2 (graph 6); we put aside for the moment the variety G and abandon the hypothesis of a production of L. By simplification, we do not consider the export costs and suppose that the firms, therefore, the nations have PDC curves identical which mix themselves. If the firms can carry out internally a monopoly price, we assume, in the first analysis, that they are able to practice a competition price in the foreign market.

In the situation of free trades where F and H can be simultaneously imported or exported, the internal maximum price of monopoly is $P(0f'')$ for F and $P(0h'')$ for H (graph 6). Beyond this, substitutable competitive goods would be imported. The effectively practiced price would be defined by the rule of equalization between the marginal cost and the marginal revenue.

Now, let's consider the case of demand for a new variety G originating from a consumer Z placed at 2 (graph 6). If the discrimination is impossible for G, the nation 1, in supplying Z for instance at the competitive price, would suffer from a decrease in its monopoly price of which the maximum would pass from $P(0f'')$ to $P(0f'')$: in fact, a part of demand for F would go to the competitive product G. This deterioration of profitability for F would be caused even when the increase in the production of the nation 1 followed supplementary selling of G would enable it attain the point of equalization between the marginal revenue and the marginal cost with a smaller quantity of F in the market 1 and, therefore, in avoiding the competition of its proper variety G, carry out a higher price.

Suppose now that 1 could discriminate her prices, that is, sell dearer her product G in the domestic market than in the market 2. The demand for F is not more necessarily affected by the production G. The nation 1, which is the only one country to produce G, is able to fix a price superior to the competitive price and comprised between $P(0g)$ and $P(0g')$. Because the nation 2 cannot practice the discrimination, it cannot propose G in its domestic market at the conditions of equally favourable global profitability: an eventual competitive price would indicate a decrease in the monopoly price of H to $P(0h'')$. The nation 2 could, of course, increase jointly the prices of H and G, but at the risk to reinforce the attract of the market of G in 2 for the nation 1.

Naturally, if 1 exports G, the monopoly gains would diminish any way. 2 could try to produce G finally. However, the opposition against 1 would have the risk be unfavourable for 2. In effect, the ≪prices-war≫ on the product G carried out by the nation 2 would cause an abrupt fall of price of the product H, even when such a policy led by 1 would not have any incidence on a variety F protected by the discrimination. On the contrary, the more the monopoly gain of 1 on G in 2 would increase, the better the monopoly gain of 2 on H would be preserved.

The nation 1 posesses, therefore, an advantage on the nation 2 which would enable it put up with relatively high costs of export.

In order that there would be trade within branch, it is necessary that there would be also the export of 2 towards 1.

Of course, the variety H can be demanded. However, if there would exist further in 1 non-satisfied demand of F, imperfect substitute, or by G, sold too dear in the domestic market (by discrimination), 2 could produce this variety and export it towards 1 in carrying out the price discrimination to protect its monopoly in H. The gains of 1 are, then, constrained because the variety F are more narrowly in competition. 1 could decide to cut down the price of G in its domestic market to compete against a part of the demand for import. In the face of changes of its demand curves, the firm can redefine its production program and, eventually, propose new varieties or sell on the previously insufficiently profitable markets. Everythings turn out to be as if the unsatisfaction of domestic consumers would attract foreign producers. These models by Professor J.M. Siroen would be very useful for the international trades of monopolistic goods and services, for example, automobiles, household appliances, machine-tools, iron and steel, various financial commodities and so forth. However, as for the econometric studies based on these models would require some further elaborations and statistical data. Consequently, in order to make empirical analysis, we intend to practice a factor analysis.

Note

(1) Jean-Marc Siroen, ≪Discrimination des prix, différenciation des produits et échange international. ≫ Revue économique—N° 3, mai 1986. pp. 489–520.

(2) Joan Violet Robinson, The Economics of Imperfect Competition, London, Macmillan, 1933.

(3) W.J. Rieber, ≪Discriminating monopoly and international trade ≫. Economic Journal, June, 1982. pp. 365–376.

(4) K. Lancaster, ≪Change and innovation in the technology of consumption ≫, American Economic Review, May, 1966. pp. 14–23 ≪A new approach to consumer theory. ≫, Journal of Political Economy. 2. April, 1966. pp. 132–157.

(5) H.E. Leland, ≪Quality choice and competition ≫, American Economic Review 2 March, 1977, pp. 127–136. M.B. Steward, ≪Monopoly and the choice of product characteristics ≫, Economic Letters 2. 1977. pp. 79–84.

(6) Edward H. Chamberlin, The Theory of Monopolistic Competition, 7th ed.; Cambridge, Mass.: Havard, 1956. K. Lancaster, ≪Intra-industry trade under perfect monopolistic competition ≫, Journal of International Economics. 10. 1980. pp. 151–175.

(7) K. Lancaster, op. cit., A.E.R. May, 1966, pp. 14–23: op. cit., J.P.E., 2. April. 1966. pp. 132–157.

(8) G.C. Archibald and G. Rosenbluth, ≪The "new" theory of consumer demand and monopolistic competition ≫. Quatarly Journal of Economics, 4, November, 1975, pp. 569–590.

(9) L.J. White, ≪Market structure and product varieties ≫, American Economic Review, 2, March, 1977. pp. 179–182.

(10) K. Lancaster, ≪Socially optimal product differentiation ≫, American Economic Review, 4, September, 1975. pp. 567–585. G.C. Archibald and G. Rosenbluth, op. cit., Q.J.E., 4, November, 1975.

(11) L.J. White, op. cit., pp. 179–182.

(12) M. Spence, ≪Product selection, fixed cost and monopolistic competition ≫, Review of Economic Studies, 134. 1976. pp. 217–236.

(13) K. Lancaster, ≪Intra-industry trade under perfect monopolistic competition ≫, Journal of International Economics, 10. 1980. pp. 151–175.

(14) P.R. Krugman, ≪Increasing returns, monopolistic competition and international trade ≫. Journal of International Economics, 9. 1979. pp. 469–479. ≪Scale economies, product differentiation and the pattern of trade ≫. American Economic Review. 5. December, 1980, pp. 950–959. ≪Intra-industry specialization and the gains from trade ≫. Journal of Political Economy. 5. 1981. pp. 959–973.

(15) H.P. Gray, ≪Two way international trade in manufactures: a theoretical underpinning ≫. Weltwirtschaftliches Archic. 1. 1973. pp. 19–39.

3. Factor analysis

According to Spearman[1] (1904), we assume the factor g correlated positively with all variates z_j $(j = 1, 2, \ldots, 6)$, and obtain the following linear model:

$$z_j = a_j g + u_j \qquad (j = 1, 2, \cdots, 6)$$

However, both z_j and g are standardized before-hand:

$$\begin{aligned} &E(Z_j) = 0, \qquad E(Z_j^2) = 1 \qquad (j = 1, 2, \cdots, 6) \\ &E(g) = 0, \qquad E(g^2) = 1 \end{aligned}$$

As for the residuals we assume:

$$E(u_j) = 0, \qquad E(u_j^2) = U_j^2$$

After the two factors theory, we call generally g general factor, u_j specific factor and a is called factor loading.

If two factors theory can be held, the correlations among factors could be written:

$$\begin{aligned} r_{guj} &= E(g \cdot u_j) = 0 \\ r_{ujuk} &= E(u_j \cdot u_k) = 0 \qquad (j \neq k) \end{aligned}$$

Therefore, the correlations among variates are:

$$\begin{aligned} r_{jk} &= E(Z_j \cdot Z_k) \\ &= E[(a_j g + u_j)(a_k g + u_k)] \\ &= E(a_j a_k g^2 + a_j g u_k + a_k g u_j + u_j u_k) \\ &= \begin{cases} a_j a_k & (j \neq k) \\ a_j^2 + U_j^2 & (j = k) \end{cases} \end{aligned}$$

When there are m variates generally, the correlation matrix composed of each correlation-coefficients among m variates as their elements is R. The matrix $\boldsymbol{R}^+$ which are replaced by a_j^2 instead of the jth diagonal elements in the matrix $\boldsymbol{R}$ is called reduced correlation matrix:

$$
\boldsymbol{R}^{+} = \begin{bmatrix}
a_1^2 & \cdots & a_1 a_j & \cdots & a_1 a_k & \cdots & a_1 a_m \\
\vdots & & \vdots & & \vdots & & \vdots \\
a_j a_1 & \cdots & a_j^2 & \cdots & a_j a_k & \cdots & a_j a_m \\
\vdots & & \vdots & & \vdots & & \vdots \\
a_k a_1 & \cdots & a_k a_j & \cdots & a_k^2 & \cdots & a_k a_m \\
\vdots & & \vdots & & \vdots & & \vdots \\
a_m a_1 & \cdots & a_m a_j & \cdots & a_m a_k & \cdots & a_m^2
\end{bmatrix}
$$

Clearly,

$$\text{Rank } (\boldsymbol{R}^{+}) = 1$$

Take the second order determinant:

$$
\begin{aligned}
D &= \begin{vmatrix} r_{ij} & r_{il} \\ r_{kj} & r_{kl} \end{vmatrix} = r_{ij} r_{kl} - r_{il} r_{kj} \\
&= a_i a_j a_k a_l - a_i a_l a_k a_j \\
&= 0
\end{aligned}
$$

This is called telrads.

Column sum T_j and total sum of T^{+} are:

$$T_j = (a_1 + a_2 + \cdots + a_m) a_j$$

$$T_{..} = (a_1 + a_2 + \cdots + a_m)^2$$

Therefore, factor loadings a_j are:

$$a_j = \frac{T_j}{\sqrt{T_{..}}}$$

The procedure by which factor loadings are obtained is called centroid method.(2)

We cannot get diagonal elements a_j^2 of $\boldsymbol{R}^{+}$ without obtaining a_j. Therefore, for example, according to the correlation matrix of Spearman, we can get a_j conveniently as $\widetilde{a}_j^2$ which are the maximum values in each column. Then, we are going to reconstruct reduced correlation matrix by the products of obtained $\hat{a}_j$ and let it be $\hat{\boldsymbol{R}}^{+}$. We can define residual matrix as $\boldsymbol{E} = \boldsymbol{R} - \hat{\boldsymbol{R}}^{+}$ etc.

Common factors

Generalizing two factors theory, when the number of orders in R^{+}

are superior to or equal to 2 ($p \geqq 2$), we consider p factors which are independent and linear each other: f_k $(k = 1, 2, \ldots, p)$:

$$z_j = a_{j1} f_1 + a_{j2} f_2 + \cdots + a_{jp} f_p + u_j \quad (j = 1, 2, \cdots, m)$$

which is constructed by Thurstone[3] (1947).

These p factors are called common factors and this is a fundamental equation of common factor analysis. Both z and f are standardized beforehand:

$$E(Z_j) = 0 \qquad E(Z_j^2) = 1,$$

$$E(f_k) = 0 \qquad E(f_k^2) = 1$$

As for u, the same case as two factors theory holds is:

$$E(u_j) = 0 \qquad E(u_j^2) = U_j^2$$

The correlation between common factors and specific factors is zero:

$$r_{fkuj} = E(f_k \cdot u_j) = 0$$

P common factors can be correlated, but it would be possible to assume conveniently:

$$r_{fkfl} = E(f_k \cdots f_l) = 0$$

In this case the correlations among the variates z are

$$\begin{aligned} r_{ij} &= E(z_i \cdot z_j) \\ &= E[(a_{i1}) f_1 + \cdots + u_i)(a_{j1} f_1 + \cdots + u_j)] \\ &= a_{i1} a_{j1} + a_{i2} a_{j2} + \cdots + a_{ip} a_{jp} \qquad (i = j) \end{aligned}$$

In the case where i is equal to j $(i = j)$:

$$\begin{aligned} 1 &= a_{jl}^2 + a_{j2}^2 + \cdots + a_{jp}^2 + u_j^2 \\ &= h_j^2 + u_j^2 \end{aligned}$$

where: $\quad h_j^2 = a_{j1}^2 + a_{j2}^2 + \cdots + a_{jp}^2$

h_j^2 is communality and u_j^2 is uniqueness.

The correlation matrix whose diagonal elements are replaced by h_j^2 is reduced correlation matrix $\boldsymbol{R}^+$ and is represented by both vector and matrix:

$$\mathbf{Z}=\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix}, \qquad \mathbf{f}=\begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_p \end{bmatrix}, \qquad \mathbf{u}=\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$\mathbf{A}=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1P} \\ a_{21} & a_{22} & \cdots & a_{2P} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mP} \end{bmatrix}$$

The fundamental equation is:

$$\boldsymbol{Z} = \boldsymbol{Af} + \boldsymbol{u}$$

The correlation matrix is $\boldsymbol{R}$:

$$\begin{aligned} \boldsymbol{R} &= E(\boldsymbol{zz'}) \\ &= \boldsymbol{A}\cdot E(\boldsymbol{ff'})\cdot \boldsymbol{A'}+\boldsymbol{uu'} \\ &= \boldsymbol{AA'}+\boldsymbol{U}^2, \end{aligned}$$

Here,

$$\boldsymbol{U}^2 = \begin{bmatrix} u_1^2 & & & 0 \\ & u_2^2 & & \\ & & \ddots & \\ 0 & & & u_n^2 \end{bmatrix}$$

Reduced correlation matrix is $\boldsymbol{R}^+$:

$$\boldsymbol{R}^+ = \boldsymbol{AA'} = \boldsymbol{R} - \boldsymbol{U}^2$$

Fundamental structure of matrix $\boldsymbol{A}$ is now:

$$\boldsymbol{A} = \boldsymbol{V}\Delta\boldsymbol{W'}$$

where $\boldsymbol{V'V} = \boldsymbol{I}_m \qquad \boldsymbol{W'W} = \boldsymbol{I}_p$

Fundamental structure of $\boldsymbol{R}^+$ is:

$$\begin{aligned} \boldsymbol{R}^+ &= \boldsymbol{AA'} \\ &= (\boldsymbol{V}\Delta\boldsymbol{W'})(\boldsymbol{W}\Delta\boldsymbol{V'}) \\ &= \boldsymbol{V}\Delta^2\boldsymbol{V'} \end{aligned}$$

Fundamental diagonal matrix of $\boldsymbol{R}^+$, Δ^2 having diagonal elements which are composed of p proper values λ_k $(k = 1, 2, \cdots, p)$ of $\boldsymbol{R}^+$:

$$\Delta^2 = \Lambda = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_p \end{bmatrix}$$

Fundamental normal orthogonal matrix $\boldsymbol{V}$ consists of proper vectors $\boldsymbol{v}_k$ which correspond to λ_k:

$$\boldsymbol{V} = [\boldsymbol{v}_1, \ \boldsymbol{v}_2, \ \cdots, \ \boldsymbol{v}_p]$$

Then,

$$\Delta = \Lambda^{\frac{1}{2}} = \begin{bmatrix} \sqrt{\lambda_1} & & & 0 \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_p} \end{bmatrix}$$

$$\boldsymbol{A} = \boldsymbol{V}\Delta$$

is the solution of $\boldsymbol{R}^+ = \boldsymbol{A}\boldsymbol{A}'$

However, the solution of $\boldsymbol{R}^+$ is not generally determined uniquely. Now, $\boldsymbol{T}$ is arbitrary normal orthogonal matrix:

$$\boldsymbol{T}'\boldsymbol{T} = \boldsymbol{I}$$

$\boldsymbol{A}$ is transformed by $\boldsymbol{T}'\boldsymbol{T} = \boldsymbol{I}$:

$$\boldsymbol{A} = \boldsymbol{B}\boldsymbol{T}'$$

Then,

$$\begin{aligned} \boldsymbol{R}^+ &= \boldsymbol{A}\boldsymbol{A}' = (\boldsymbol{B}\boldsymbol{T}')(\boldsymbol{B}\boldsymbol{T}')' \\ &= \boldsymbol{B}\boldsymbol{B}' \end{aligned}$$

Therefore, $\boldsymbol{B}$ is also the solution.

In this case the fundamental equation is:

$$\boldsymbol{z} = \boldsymbol{B}\boldsymbol{f}^* + \boldsymbol{u} \qquad \text{where, } \boldsymbol{f}^* = \boldsymbol{T}\boldsymbol{f}$$

The transformation by $\boldsymbol{T}$ is called rotation in factor space.

Factor loading matrix $\boldsymbol{A}$ is determined by fundamental structure of reduced correlation matrix $\boldsymbol{R}^+$ and therefore, $\boldsymbol{A}$ is obtained by the solution of proper values problem such as the matrix of elements composed by proper vectors. The matrix $\boldsymbol{A}$ calculated in this way is called principal axis solution or principal divisor axis.

If one of solutions is derived from $\boldsymbol{R}^{+}$, rotation would be carried out according to some established convenient rule:

$$\boldsymbol{B} = \boldsymbol{AT}$$

is obtained, and then, easily interprated solution would be often factor loading matrix. Factor scores could be derived from this obtained factor loading matrix.

Estimation of communality and the number of factors:

The scope of variations in communality h_j^2 is:

$$0 \leqq h_j^2 \leqq 1$$

Both one (1) and (0) are used instead of communality. Using are (1) instead of communality is the same not to reduce the diagonal elements of correlation matrix beforehand by uniqueness. This is the same analysis called principal component analysis.

It is assumed formally that:

$$\tilde{\boldsymbol{R}}_1^{+} = \boldsymbol{R}$$

The number of orders p_1 is in general the same number of variates m and m proper values are all positive. Then, it is natural to choose some proper vectors in order of magnitudes of proper values according to some rules to get some interpretations on data.

If diagonal elements are zero (0), reduced correlation matrix is:

$$\tilde{\boldsymbol{R}}_2^{+} = \boldsymbol{R} - \boldsymbol{I}$$

It would perhaps be said that the uniqueness is overestimated. The proper vectors in $\tilde{\boldsymbol{R}}_2^{+}$ are the same of those in $\tilde{\boldsymbol{R}}_1^{+}$ and the proper values of $\tilde{\boldsymbol{R}}_2^{+}$ are smaller of one than those of $\tilde{\boldsymbol{R}}_1^{+}$ in each. p_2, which represents the number of positive proper values in $\tilde{\boldsymbol{R}}_2^{+}$ seems to be the smallest among all the other methods.

The most frequently used method for calculation by computers takes square of multiple correlation (SMC) between each variate and other variates for the estimates of h_j^2. SMC method is based on the image analysis method by Guttman[4] (1956) which would divide correlations (variations) between images and anti-images instead of the division of those between communality and uniqueness. Image is the part of variations in certain variate z_j explained by other $(m-1)$ variates and on the other hand, anti-image is the part unexplained by these $(m-1)$ variates. R_j^2 is the ratio of the explained part and is the largest among the estimates of lower limit on communality. Reduced correlation matrix which has the diagonal elements of R_j^2 is written:

$$\tilde{\boldsymbol{R}}_3^+ = \boldsymbol{R} - \boldsymbol{S}^2$$

where, $\boldsymbol{S}^2$ is diagonal matrix composed by elements of inverses of diagonal elements in inverse matrix of $\boldsymbol{R}$:

$$\boldsymbol{S}^2 = [\text{diag } \boldsymbol{R}^{-1}]^{-1}$$

p_3 is the number of positive proper values in $\tilde{\boldsymbol{R}}_3^+$ and then:

$$p_2 \leqq p_3 \leqq p \leqq p_1 = m$$

The number of factors, p, must be given as that of positive proper values, however, it is not necessary appropriate to estimate p by counting the number of positive proper values of $\tilde{\boldsymbol{R}}^+$ without understanding good approximate of $\tilde{\boldsymbol{R}}^+$ on $\boldsymbol{R}$ or not.

There are several methods by which can determine the number of p. First of all, we put proper values in order of cardinal numbers and then, if there is large difference of magnitudes among proper values put in cardinal order, we are used to eliminate the factors which have smaller proper values than before the large difference.

Secondly, we take larger number of factors than we can consider beforehand and after the rotation, we should make its rule only to take up the factors which are able to have interpretations on them. Thirdly, it is often to abandon the proper values inferior to one (1) when we try to compute proper values without reducing the estimates of uniqueness from correlation matrix. Fourthly, as for $\tilde{\boldsymbol{R}}^+$ with some reduction, we try to eliminate some factors in similar convenient way.

However, these procedures are not based on some firm theoretical foundations, so analysts would decide arbitrarily which methods are chosen in the factor analysis.

If the number of factors, p, are exactly estimated and the estimate of factor loading matrix ($\hat{\boldsymbol{A}}$ of $\boldsymbol{A}$) is obtained, then reduced correlation matrix $\boldsymbol{R}^+$ is restored by the following equation:

$$\hat{\boldsymbol{R}}^+ = \hat{\boldsymbol{A}}\hat{\boldsymbol{A}}',$$

residual matrix

$$\boldsymbol{E} = \boldsymbol{R} - \hat{\boldsymbol{R}}^+$$

has almost all elements equal to zero (0) except diagonal elements that must be between zero (0) and one (1). If common factor analysis would be completely valid, all non-diagonal elements of $\boldsymbol{E}$ should be zero (0). Paying attention to this respect, it would be possible to eliminate the arbitrariness of previous factor analysis in carrying out the least square method, which obtains the solution on the rule of minimizing a sum of squares of non-diagonal elements at the beginning, or the maximum likelihood method, which try to get the solution in assuming that all variates are normally distributed and to evaluate the fitness of this estimations.

Varimax method (Orthogonal rotation)

When the number of variates and factors increase, it is not easy to make an oblique rotation into simple structure according to the rule of Thurstone. This is because of the lack of strictness in the rule and of very quantitative computations. Under the constraints of which reference—axes are orthogonal each other and analytical rules to be differentialable beforehand, it would be possible to obtain fairly good solution to make an optimum rotation against the rule.

$$V_j = \frac{1}{m}\sum_{i=1}^{m} b_{ij}^4 - \frac{1}{m^2}\left(\sum_{i=1}^{m} b_{ij}^2\right)^2$$

This is conveniently multiplied by k and summed totally:

$$V = m\sum_{j=1}^{p} V_j = \sum_{j=1}^{p}\sum_{i=1}^{m} b_{ij}^4 - \frac{1}{m}\sum_{j=1}^{p}\left(\sum_{i=1}^{m} b_{ij}^2\right)^2$$

If V is maximized and rotated, factor loadings will be in majority near zero (0) and relatively large in absolute value, therefore a part of Thurstone [5] rule will be satisfied. V is the varimax criterium.

In order to get T to maximize V, the constraint is:

$$u_{jh} = \sum_{k=1}^{p} t_{kj} t_{kh} = \begin{cases} 1 \cdots\cdots & j = h \\ 0 \cdots\cdots & j \neq h \end{cases} \quad (j, h = 1, 2, \cdots, p)$$

and the following equation can be solved.

$$\frac{\partial V}{\partial t_{qs}} - \frac{\partial}{\partial t_{qs}} \sum_{j=1}^{p} 2\lambda_{jj} \left(\sum_{k=1}^{p} -1 \right)$$

$$- \frac{\partial}{\partial t_{qs}} \sum_{j=1}^{p} \sum_{k=j+1}^{p} 2\lambda_{jh} \sum_{h=1}^{p} t_{kj} t_{kh} = 0 \quad (q, s = 1, 2, \cdots, p)$$

$$\frac{\partial b_{ij}}{\partial t_{qs}} = \frac{\partial}{\partial t_{qs}} \sum_{k=1}^{p} a_{ik} t_{kj} = \begin{cases} 0 & \cdots\cdots \quad j \neq S \\ a_{iq} & \cdots\cdots \quad j = S \end{cases}$$

Then, the first term:

$$\frac{\partial V}{\partial t_{qs}} = 4 \sum_{i=1}^{m} b_{is}^3 a_{iq} - \frac{4}{m} \sum_{i=1}^{m} b_{is}^2 \sum_{i=1}^{m} b_{is} a_{iq}$$

The second term:

$$\frac{\partial}{\partial t_{qs}} \sum_{j=1}^{p} 2\lambda_{jj} \left(\sum_{k=1}^{p} t_{kj}^2 - 1 \right) = 4\lambda_{ss} t_{qs}$$

The third term:

$$\frac{\partial}{\partial t_{qs}} \sum_{j=1}^{p} \sum_{h=j+1}^{p} 2\lambda_{jh} \sum_{k=1}^{p} t_{kj} t_{kh} = \sum_{h=s+1}^{p} 2\lambda_{sh} t_{qh}$$

Therefore,

$$\sum_{i=1}^{m} b_{is}^3 a_{iq} - \frac{1}{m} - \sum_{i=1}^{m} b_{is}^2 \sum_{i=1}^{m} b_{is} a_{iq} - \lambda_{ss} t_{qs}$$

$$-\frac{1}{2} \sum_{h=s+1}^{p} \lambda_{sh} t_{qh} = 0$$

Now,

$$\boldsymbol{C} = [b_{ij}^3], \qquad \boldsymbol{D} = \frac{1}{m} \cdot [\text{diag } \boldsymbol{b}'\boldsymbol{B}]$$

$$\Gamma = \begin{bmatrix} \lambda_{11} & \frac{1}{4}\lambda_{12} & \cdots & \frac{1}{4}\lambda_{1p} \\ \frac{1}{4}\lambda_{12} & \lambda_{22} & \cdots & \frac{1}{4}\lambda_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{4}\lambda_{1p} & \frac{1}{4}\lambda_{2p} & \cdots & \lambda_{pp} \end{bmatrix}$$

Differential equation is:

$$\boldsymbol{A}'\boldsymbol{C} - \boldsymbol{A}'\boldsymbol{B}\boldsymbol{D} - \boldsymbol{T}\Gamma = 0$$

$$T^{-1} = T'$$

Then,

$$\Gamma = T'A' \ (C - BD).$$

$\Gamma = \Gamma'$ Therefore,

$$T'A'(C - BD) = (C - BD)'AT.$$

$$G = A'(C - BD)$$

$$T'G = G'T.$$

Multiply both hands by T from the left side:

$$G = TG'T.$$

Multiplication of T' from the right side:

$$GT' = TG'.$$

Multiplication of TG' from the right side:

$$GG' = [TG']^2.$$

When proper values problem of GG' is solved, its fundamental structure is:

$$GG' = Q\Delta^2 Q'$$

The square root of matrix GG' is given:

$$[GG']^{\frac{1}{2}} = Q\Delta Q'$$

Therefore,

$$TG' = [GG']^{\frac{1}{2}}$$

$$\therefore \quad T = [GG']^{-\frac{1}{2}} G$$

On the basis of the above equation, T would be obtained by sequential qualification method. Initial matrix of T is $T_{(0)} = I$ and T of the gth repetitive computation is $T_{(g)}$:

$$B_{(g)} = AT_{(g)},$$

$$C_{(g)} = [b_{ij}^3]_{(g)},$$

$$D_{(g)} = \frac{1}{m}[\text{diag } \boldsymbol{B}'\boldsymbol{B}],$$
$$\boldsymbol{G}_{(g)} = \boldsymbol{A}'(\boldsymbol{C}_{(g)} - \boldsymbol{B}'_{(g)}\boldsymbol{D}_{(g)}),$$
$$\boldsymbol{T}_{(g+1)} = [\boldsymbol{G}_{(g)}\boldsymbol{G}'_{(g)}]^{-\frac{1}{2}}\boldsymbol{G}_{(g)}$$

Therefore $\boldsymbol{T}_{g+1}$ is given.(6)

This is the summary of theoretical frameworks on the factor analysis of which the least squares method would be utilized to compute factor loadings of sixteen Japan's import-export functions both for the long term and for the short term based on the statistical data.

Note

(1) Kazuo Nakatani, Tahenryo Kaiseki (Multivariates Analysis), Shinyo-Sha, Tokyo, 1978. pp. 26–57.

(2) K. Nakatani, op. cit., pp. 27–29.

(3) L.L. Thurstone, Multiple Factor Analysis, Chicago, University of Chicago Press, 1947.

(4) L. Guttman, "'Best Possible' Systematic Estimates of Communalities," Psychometrika, 21, 1956. pp. 273–285.

(5) L.L. Thurstone, op. cit., 1947.

(6) K. Nakatani, op. cit., pp. 26–52.

4.

Empirical estimations on the Japan's import-export functions by the least squares method of factor analysis(1) could be represented in the following tables 4.1–4.8. We can find out 4 proper values superior to about 0.8, and therefore, we should be able to show the variations of each variables from about 80% to 90% by 4 factors of which their factor loadings represented by the varimax rotation are indicated and, then, the largest values of factor loading are clearly shown by contained recutangulars.(2) The definition of variables are almost all common to those of which are from No. 1 variable to No. 17 for the long term and from No. 1 to No. 18 for the short term, except two Japan's import-export functions (Fixed Exchange-Rate System: 1956I–1971II) from No. 1 to No. 14 for the long term and from No. 1 to No. 15 for the short term.(3) The definitions of each variable are such as follows:

Definitions of variables

$X(1)$: average of relative wholesale price index in respective two countries between the current and the previous quarterly periods.

$X(2)$: average of relative wholesale price index in respective two countries between the second and the third previous quarterly periods.

$X(3)$: average of relative wholesale price index in respective two countries from the fourth to the tenth previous quarterly periods.

$X(4)$: average of relative exchange-rates in respective two countries between the current and the previous quarterly periods.

$X(5)$: average of relative exchange-rates in respective two countries between the second and the third previous quarterly periods.

$X(6)$: average of relative exchange-rates in respective two countries from the fourth to the tenth previous quarterly periods.

$X(7)$: average of industrial production index in its country (import) or in its partner country (export) between the current and the previous quarterly periods.

$X(8)$: average of industrial production index in its country (import) or in its partner country (export) between the second and the third quarterly periods.

$X(9)$: average of industrial production index in its country (import) or in its partner country (export) from the fourth to the tenth previous quarterly periods.

$X(10)$: relative seasonal dummy variable for the first quarterly period.

$X(11)$: relative seasonal dummy variable for the second quarterly period.

$X(12)$: relative seasonal dummy variable for the third quarterly period.

$X(13)$: time trend variable of the first order.

$X(14)$: time trend variable of the second order.

$X(15)$: rate of covering rate in the previous period.

$X(16)$: rate of operating capacity.

$X(17)$: rate of inventory of imported raw materials (import).

$X(17)$: rate of inventory of manufactured products (export).

$X(18)$: real volume of Japan's import in term of exchange-rate (import function).

$X(18)$: real volume of Japan's export in term of exchange-rate[4] (export function).

However, Japan's import and export function with the U.S. in the fixed exchange-rate system from 1956I to 1971II had the constant exchange-rate of dollar to yen, and, therefore, had only 14 variables for the long term and 15 variables for the short term. This is the reason why the definition of variables in two Japan's import and export functions which have not three variables: $X(4)$, $X(5)$ and $X(6)$, and other variables are exactly the same, but each number of variables is smaller of three than the above definition of variables after the variable $X(3)$, for example, variables $X(4)$ in Japan's import and export functions of the Fixed Exchange-Rate System from 1956I to 1971II belong to the variable $X(7)$ in the above definition of variables and so forth.

As for the interpretations on the results of the least squares method of factor analysis from the economic analysis are briefly represented as follows. The first factor can contains the majorities of all economic variables, especially, in 4.1, 4.2, 4.4, 4.6 and 4.8. Relative seasonal dummy variables are contained in the third or the second factors concentratedly in all three variables of one set. Some economic variables have their largest factor loadings in the second, the third and the fourth factors scatteredly. However, the fourth factor has the smallest number of economic variables with the largest factor loadings.

Note

(1) *JMA* Research Inc. *JRI* Personal Computer Soft-Ware Library, Factor Analysis and Principal Component Analysis System. *V*2.0 1987. 6066. *MS/DOS*.

(2) Statistical data; International Financial Statistics, January 1952–April 1988. Bank of Japan., The Bulletin of Economic Statistic, January 1953–April 1988.

(3) The estimations of multi-variate regression analysis on all sixteen Japan's import and export functions for both the long term and the short term are computed in other articles: See S. Suwa, "French Export-Function of Voluntary Restrictions and the Japan's Export-Function towards the France and the U.S.," The Shakai Kagaku Tôkyû (The Social Sciences Review), Vol. 33 No. 1 *XCV*, Institute of Social Sciences, Waseda University, September 1987. S. Suwa, "The Exchange-Control Model in France and Estimations of Japan's import functions of the *CES* type from the U.S. and the France and Economic Policies," The Waseda Economic Papers, Graduate School of Economics, Waseda University, No. 26 1987. S. Suwa, "Les functions d'importation en téchnologie de la France et fonctions d'importation japonaises de la France et des E.U.A. et les politiques économiques," Le Bulletin de la Société Franco—Japonaise des Siences Economiques, No. 11. Septembre 1988.

(4) For the definitions of variables in detail, see S. Suwa in The Waseda Journal of Political Science and Economics, No. 292–293, March, 1988.

Least-squares Method

4.1 Japan's import function from the U.S.
(Fixed Exchange-Rate System 1956I–1971II)

communality
MFUL (Long term)

<<Factor loadings by varimax rotation>>

	1	2	3	4	communality
X(1)	-0.0713	0.8009	0.0298	-0.0064	0.6475
X(2)	-0.0816	0.9664	-0.0128	0.4016	1.1021
X(3)	-0.0136	0.1547	-0.0226	0.6262	0.4167
X(4)	1.0015	-0.0494	-0.0122	-0.0221	1.0060
X(5)	0.9940	-0.0549	-0.0136	-0.0632	0.9952
X(6)	0.9809	-0.0148	-0.0091	-0.1456	0.9837
X(7)	-0.0064	0.0646	0.6413	-0.1456	0.9837
X(8)	0.0018	-0.0357	0.7304	0.0727	0.5400
X(9)	-0.0200	-0.0401	0.7277	0.0515	0.5342
X(10)	0.9962	-0.0033	-0.0095	-0.1243	1.0080
X(11)	0.9680	0.0168	-0.0052	-0.1961	0.9741
X(12)	0.4800	0.4739	-0.1429	-0.4118	0.6449
X(13)	0.9439	0.0582	-0.0061	0.0791	0.9006
X(14)	0.7793	-0.0882	-0.0151	0.2461	0.6759
proper value	6.6236	1.8481	1.4970	0.8923	10.8610

communality
MFUS (Short term)

<<Factor loadings by varimax rotation>>

	1	2	3	4	communality
X(1)	-0.0857	0.8154	0.0337	0.0003	0.6733
X(2)	-0.0999	0.9370	-0.0132	0.3709	1.0257
X(3)	-0.0057	0.1668	-0.0260	0.6460	0.4459
X(4)	1.0005	-0.0246	-0.0182	-0.0441	1.0040
X(5)	0.9944	-0.0327	-0.0192	-0.0815	0.9969
X(6)	0.9784	0.0047	-0.0142	-0.1634	0.9842
X(7)	-0.0107	0.0625	0.6361	-0.1318	0.4260
X(8)	0.0096	-0.0350	0.7336	0.0752	0.5452
X(9)	-0.0136	-0.0397	0.7281	0.0531	0.5347
X(10)	0.9924	0.0182	-0.0150	-0.1443	1.0062
X(11)	0.9661	0.0325	0.0008	-0.2087	0.9779
X(12)	0.4545	0.4820	-0.1469	-0.4264	0.6423
X(13)	0.9520	0.0788	-0.0095	0.0729	0.9179
X(14)	0.7836	-0.0618	-0.0204	0.2186	0.6660
X(15)	0.9397	-0.2410	0.0478	0.0582	0.9468
proper value	7.4930	1.8809	1.5004	0.9187	11.7930

4.2 Japan's import function from the U.S.
(Flexible Exchange-Rate System 1971III–1986IV)

<<Factor loadings by varimax rotation>> MFUL communality (Long term)

	1	2	3	4	communality
X(1)	0.9173	–0.1054	–0.0049	–0.2691	0.9249
X(2)	0.9302	–0.2132	–0.0255	–0.1758	0.9423
X(3)	0.9129	–0.1278	0.0459	–0.1744	0.8822
X(4)	0.7983	0.1020	–0.0497	0.0023	0.6501
X(5)	0.8320	0.1284	–0.0500	0.2352	0.7665
X(6)	0.8508	0.3189	0.0013	0.2751	0.9013
X(7)	0.9844	–0.0612	–0.0074	–0.0336	0.9740
X(8)	0.9731	0.0519	0.0009	–0.0596	0.9531
X(9)	0.9327	0.2590	–0.0016	–0.1820	0.9701
X(10)	–0.0156	0.0194	0.6923	–0.0133	0.4800
X(11)	0.0149	–0.0153	0.7441	0.1290	0.5708
X(12)	–0.0117	–0.0080	0.7007	0.0097	0.4913
X(13)	0.9618	0.1982	–0.0041	–0.1940	1.0020
X(14)	0.9312	0.0330	–0.0065	–0.3563	0.9952
X(15)	0.3149	–0.1485	–0.1541	–0.6243	0.5348
X(16)	0.0648	–0.7279	–0.0006	0.0412	0.5358
X(17)	0.3217	0.8620	–0.0066	0.3707	0.9841
proper value	9.3788	1.6111	1.5556	1.0129	13.5584

<<Factor loadings by varimax rotation>> MFUS communality (Short term)

	1	2	3	4	communality
X(1)	0.9424	–0.0838	0.0066	0.2063	0.9376
X(2)	0.9492	–0.1935	–0.0233	0.0701	0.9437
X(3)	0.9378	–0.1765	0.0257	–0.1016	0.9217
X(4)	0.7725	0.2822	–0.0034	0.2672	0.7478
X(5)	0.7818	0.3132	–0.0210	–0.0293	0.7106
X(6)	0.8019	0.3946	–0.0157	–0.3414	0.9157
X(7)	0.9837	–0.0261	–0.0100	–0.0860	0.9758
X(8)	0.9760	0.0561	–0.0099	–0.1304	0.9729
X(9)	0.9479	0.2154	–0.0191	–0.0758	0.9510
X(10)	–0.0124	0.0090	0.6903	–0.0129	0.4770
X(11)	–0.0028	0.0083	0.7164	–0.0948	0.5222
X(12)	–0.0087	–0.0296	0.7059	–0.0643	0.5034
X(13)	0.9761	0.1842	–0.0101	0.0121	0.9869
X(14)	0.9727	–0.0312	–0.0156	0.0914	0.9557
X(15)	0.3917	–0.2147	–0.1365	0.6842	0.6863
X(16)	0.0697	–0.6246	0.0149	0.0863	0.4027
X(17)	0.2576	0.8357	0.0066	–0.2003	0.8050
X(18)	0.2377	0.1553	0.1195	–0.7358	0.6364
proper value	9.5816	1.6540	1.5241	1.3557	14.0524

4.3 Japan's import function from the France
(Fixed Exchange-Rate System 1956I–1971II)

communality

<<Factor loadings by varimax rotation>> MFUL (Long term)

	1	2	3	4	communality
X(1)	0.4472	0.0046	–0.8354	0.1909	0.9343
X(2)	0.4507	–0.0247	–0.8618	0.1531	0.9699
X(3)	0.5994	0.0093	–0.7338	0.0404	0.8995
X(4)	–0.4294	0.0116	0.8498	–0.1430	0.9270
X(5)	–0.4405	0.0087	0.8679	–0.0841	0.9545
X(6)	–0.5601	–0.0067	0.7438	0.0026	0.8670
X(7)	0.8600	0.0022	–0.5031	0.1046	1.0037
X(8)	0.8494	0.0004	–0.5087	0.1103	0.9925
X(9)	0.8263	0.0031	–0.5279	0.1419	0.9817
X(10)	–0.0172	0.6411	0.0256	0.1588	0.4372
X(11)	0.0142	0.7302	0.0028	–0.1330	0.5511
X(12)	0.0080	0.7111	–0.0196	–0.0850	0.5133
X(13)	0.8603	0.0020	–0.4829	0.1516	0.9963
X(14)	0.9022	0.0127	–0.3127	0.1866	0.9467
X(15)	0.2889	–0.0933	–0.2797	1.2428	1.7149
X(16)	0.7189	–0.0033	–0.2736	0.0116	0.5918
X(17)	0.7535	–0.0156	–0.4770	0.0965	0.8048
proper value	6.3210	1.4600	5.5109	1.7943	15.0861

communality

<<Factor loadings by varimax rotation>> MFUS (Short term)

	1	2	3	4	communality
X(1)	0.4278	0.0072	–0.8451	0.1964	0.9358
X(2)	0.4318	–0.0218	–0.8706	0.1558	0.9691
X(3)	0.5869	0.0126	–0.7429	0.0390	0.8981
X(4)	–0.4085	0.0088	0.8599	–0.1461	0.9277
X(5)	–0.4258	0.0066	0.8730	–0.0833	0.9505
X(6)	–0.5518	–0.0093	0.7492	0.0063	0.8659
X(7)	0.8499	0.0063	–0.5214	0.1090	1.0061
X(8)	0.8416	0.0041	–0.5250	0.1147	0.9971
X(9)	0.8153	0.0071	–0.5455	0.1481	0.9843
X(10)	–0.0373	0.6467	0.0143	0.1692	0.4485
X(11)	0.0098	0.7295	0.0018	–0.1364	0.5509
X(12)	0.0121	0.7086	–0.0137	–0.0877	0.5101
X(13)	0.8470	0.0065	–0.5039	0.1587	0.9966
X(14)	0.8960	0.0173	–0.3326	0.1963	0.9523
X(15)	0.2828	–0.0941	–0.2984	1.1847	1.5760
X(16)	0.7244	–0.0013	–0.2804	0.0108	0.6036
X(17)	0.7463	–0.0125	–0.4910	0.1011	0.8085
X(18)	0.9047	–0.0734	–0.2878	0.0603	0.9103
proper value	6.9640	1.4682	5.7834	1.6757	15.8913

4.4 Japan's import function from the France
(Flexible Exchange-Rate System 1971I–1986IV)

<<Factor loadings by varimax rotation>>

communality
MFUL (Long term)

	1	2	3	4	communality
X(1)	0.9562	−0.1541	−0.0214	0.2352	0.9938
X(2)	0.9455	−0.1655	−0.0261	0.2101	0.9662
X(3)	0.8967	−0.0685	−0.0242	0.2249	0.8600
X(4)	−0.9663	−0.0149	0.0358	−0.1591	0.9605
X(5)	−0.9761	0.0198	0.0176	−0.1517	0.9765
X(6)	−0.7146	0.0785	−0.1580	0.1210	0.5565
X(7)	0.9654	0.0323	−0.0306	−0.2412	0.9920
X(8)	0.3207	0.2793	0.0886	0.0624	0.1926
X(9)	0.9063	0.3746	−0.0182	−0.0282	0.9628
X(10)	−0.0116	0.0127	0.6820	0.0074	0.4655
X(11)	0.0122	−0.0151	0.7352	−0.1368	0.5597
X(12)	−0.0023	0.0226	0.7170	0.0345	0.5158
X(13)	0.9394	0.2978	−0.0275	−0.0719	0.9771
X(14)	0.9879	0.1323	−0.0309	0.0938	1.0032
X(15)	0.4853	−0.1328	−0.1868	0.5099	0.5481
X(16)	0.1391	−0.7488	0.0055	−0.0963	0.5893
X(17)	0.1268	0.8573	−0.0163	−0.4018	0.9127
proper value	8.9953	1.7024	1.5941	0.7405	13.0322

<<Factor loadings by varimax rotation>>

communality
MFUS (Short term)

	1	2	3	4	communality
X(1)	0.9502	−0.1801	−0.0333	−0.2429	0.9954
X(2)	0.9425	−0.2036	−0.0415	−0.1808	0.9641
X(3)	0.9012	−0.1196	−0.0438	−0.1521	0.8515
X(4)	−0.9609	−0.0069	0.0382	0.2138	0.9706
X(5)	−0.9717	0.0379	0.0224	0.1857	0.9806
X(6)	−0.7057	0.0940	−0.1647	−0.0524	0.5368
X(7)	0.9597	0.0255	−0.0241	0.1962	0.9608
X(8)	0.3439	0.2154	0.0717	0.0753	0.1755
X(9)	0.9237	0.3275	−0.0248	0.0926	0.9696
X(10)	−0.0092	0.0166	0.6786	−0.0357	0.4622
X(11)	0.0053	0.0030	0.7665	0.0129	0.5877
X(12)	0.0075	−0.0056	0.7008	0.0073	0.4913
X(13)	0.9456	0.2800	−0.0241	0.0484	0.9754
X(14)	0.9912	0.1046	−0.0363	−0.0956	1.0040
X(15)	0.4625	−0.0699	−0.1952	−0.6949	0.7398
X(16)	0.1056	−0.6916	0.0042	0.0193	0.4899
X(17)	0.1533	0.8582	0.0049	0.3936	0.9150
X(18)	0.0298	0.1482	−0.1186	0.7080	0.5381
proper value	8.9876	1.5842	1.6335	1.4029	13.6083

4.5 Japan's export function towards the U.S. (Fixed Exchange-Rate System 1956I–1971II)

communality
MFUL (Long term)

<<Factor loadings by varimax rotation>>

	1	2	3	4	communality
X(1)	0.0346	−0.0520	0.6533	−0.0011	0.4307
X(2)	0.0982	0.0375	0.9357	0.3795	1.0307
X(3)	−0.0407	0.0547	0.1244	0.6424	0.4328
X(4)	0.9927	−0.0022	0.0451	0.0113	0.9876
X(5)	0.9966	−0.0043	0.0339	0.0431	0.9963
X(6)	0.9936	−0.0032	0.0159	0.0618	0.9913
X(7)	0.0028	0.6172	−0.1035	0.1988	0.4312
X(8)	−0.0018	0.7253	0.0395	−0.0582	0.5310
X(9)	−0.0189	0.7320	0.0565	−0.0466	0.5415
X(10)	0.9963	−0.0116	−0.0326	−0.0121	0.9939
X(11)	0.9807	0.0036	−0.0503	0.0521	0.9671
X(12)	0.4706	−0.1416	−0.5142	0.3355	0.6184
X(13)	0.7694	−0.0168	0.0275	−0.3387	0.7078
X(14)	0.9671	−0.0130	−0.0221	0.0067	0.9361
proper value	6.6820	1.4705	1.6060	0.8378	10.5963

communality
MFUS (Short term)

<<Factor loadings by varimax rotation>>

	1	2	3	4	communality
X(1)	0.0264	−0.0550	0.6699	−0.0009	0.4525
X(2)	0.0935	0.0372	0.9093	0.3658	0.9708
X(3)	−0.0425	0.0567	0.1300	0.6597	0.4571
X(4)	0.9919	0.0044	0.0569	0.0057	0.9871
X(5)	0.9946	0.0028	0.0464	0.0370	0.9928
X(6)	0.9912	0.0042	0.0286	0.0554	0.9863
X(7)	0.0007	0.6136	−0.1002	0.1944	0.4243
X(8)	−0.0097	0.7297	0.0395	−0.0593	0.5377
X(9)	−0.0194	0.7239	0.0520	−0.0444	0.5291
X(10)	0.9966	−0.0049	−0.0221	−0.0185	0.9941
X(11)	0.9791	0.0109	−0.0384	0.0457	0.9623
X(12)	0.4845	−0.1410	−0.5127	0.3278	0.6249
X(13)	0.7680	−0.0125	0.0333	−0.3442	0.7096
X(14)	0.9701	−0.0071	−0.0129	0.0005	0.9413
X(15)	0.9855	−0.0889	−0.1188	0.0181	0.9936
proper value	7.6560	1.4689	1.5933	0.8454	11.5635

4.6 Japan's export function towards the U.S.
(Flexible Exchange-Rate System 1971III–1986IV)

<<Factor loadings by varimax rotation>> communality MFUL (Long term)

	1	2	3	4	communality
X(1)	−0.9372	0.1799	0.0070	−0.0974	0.9202
X(2)	−0.9454	0.2282	0.0238	0.0215	0.9469
X(3)	−0.9422	0.1543	0.0241	0.0831	0.9191
X(4)	0.5691	−0.0403	−0.0795	0.4220	0.5099
X(5)	0.0444	0.0638	0.0660	0.4234	0.1897
X(6)	0.8186	0.4753	0.0204	−0.1121	0.9089
X(7)	0.9022	0.0569	−0.0101	0.2373	0.8736
X(8)	0.9130	0.1636	0.0091	0.1952	0.8986
X(9)	0.9230	0.3149	−0.0199	0.1212	0.9661
X(10)	−0.0126	−0.0240	0.6876	0.0353	0.4748
X(11)	0.0085	0.0265	0.7422	−0.1243	0.5671
X(12)	−0.0299	0.0020	0.7076	0.1624	0.5280
X(13)	0.9820	0.1410	−0.0180	0.1355	1.0028
X(14)	0.9711	−0.0795	−0.0329	0.1053	0.9615
X(15)	0.3789	−0.4476	−0.2126	0.3505	0.5120
X(16)	0.0526	−0.5734	0.0347	−0.0940	0.3416
X(17)	0.5986	0.6862	−0.0148	0.0697	0.8343
proper value	8.5681	1.4966	1.5853	0.7050	12.3550

<<Factor loadings by varimax rotation>> communality MFUS (Short term)

	1	2	3	4	communality
X(1)	−0.9380	0.1213	−0.0001	−0.1575	0.9193
X(2)	−0.9534	0.2031	0.0238	−0.0231	0.9512
X(3)	−0.9436	0.1341	0.0238	−0.0025	0.9089
X(4)	0.5743	0.0498	−0.0640	0.3094	0.4321
X(5)	0.0514	0.1279	0.0791	0.2764	0.1016
X(6)	0.8096	0.4548	0.0178	−0.2446	0.9224
X(7)	0.9071	0.1138	−0.0007	0.1540	0.8596
X(8)	0.9150	0.2086	0.0170	0.0902	0.8891
X(9)	0.9081	0.3644	−0.0102	0.0586	0.9610
X(10)	−0.0161	−0.0180	0.6897	0.0519	0.4789
X(11)	0.0043	−0.0092	0.7381	−0.1552	0.5689
X(12)	−0.0295	0.0231	0.7128	0.1198	0.5238
X(13)	0.9714	0.2044	−0.0068	0.1454	1.0065
X(14)	0.9661	−0.0127	−0.0226	0.1915	0.9707
X(15)	0.3759	−0.3238	−0.1954	0.6086	0.6548
X(16)	0.0837	−0.6082	0.0210	−0.0756	0.3831
X(17)	0.5821	0.7044	−0.0045	−0.0168	0.8355
X(18)	0.8989	−0.1108	−0.0626	0.3125	0.9219
proper value	9.3250	1.5150	1.5838	0.8656	13.2894

4.7 Japan's export function towards the France
(Fixed Exchange-Rate System 1956I–1971II)

<<Factor loadings by varimax rotation>> MFUL communality (Long term)

	1	2	3	4	communality
X(1)	−0.3913	0.0781	0.8173	−0.1903	0.8633
X(2)	−0.4816	0.0288	0.8484	−0.1436	0.9731
X(3)	−0.6392	−0.0863	0.6428	−0.0169	0.8295
X(4)	−0.4441	0.0182	0.8439	−0.1373	0.9286
X(5)	−0.4695	0.0182	0.8540	−0.0769	0.9560
X(6)	−0.5932	−0.0116	0.7084	0.0143	0.8541
X(7)	0.8629	−0.0013	−0.4699	0.1354	0.9837
X(8)	0.8441	−0.0200	−0.4996	0.0992	0.9724
X(9)	0.8289	−0.0061	−0.5365	0.1277	0.9913
X(10)	−0.0102	0.6344	0.0294	0.1671	0.4314
X(11)	0.0247	0.7275	0.0049	−0.1309	0.5470
X(12)	−0.0133	0.7247	−0.0007	−0.1046	0.5362
X(13)	0.8825	−0.0087	−0.4480	0.1417	0.9996
X(14)	0.9151	−0.0065	−0.2752	0.1768	0.9445
X(15)	0.2909	−0.1021	−0.2709	1.2287	1.6780
X(16)	0.7152	0.0009	−0.2528	0.0150	0.5756
X(17)	0.8467	−0.0130	−0.4737	0.1273	0.9576
proper value	6.6375	1.4832	5.1414	1.7599	15.0220

<<Factor loadings by varimax rotation>> MFUS communality (Short term)

	1	2	3	4	communality
X(1)	−0.3597	0.0767	0.8325	−0.1884	0.8638
X(2)	−0.4504	0.0285	0.8644	−0.1402	0.9705
X(3)	−0.6254	−0.0856	0.6580	−0.0106	0.8315
X(4)	−0.4014	0.0171	0.8684	−0.1346	0.9336
X(5)	−0.4374	0.0180	0.8697	−0.0731	0.9534
X(6)	−0.5733	−0.0108	0.7237	0.0203	0.8530
X(7)	0.8465	−0.0003	−0.5036	0.1285	0.9866
X(8)	0.8249	−0.0193	−0.5336	0.0920	0.9741
X(9)	0.8086	−0.0055	−0.5694	0.1210	0.9927
X(10)	0.0000	0.6354	0.0369	0.1695	0.4338
X(11)	0.0176	0.7264	−0.0009	−0.1312	0.5452
X(12)	−0.0174	0.7224	−0.0025	−0.1045	0.5330
X(13)	0.8647	−0.0076	−0.4846	0.1345	1.0007
X(14)	0.9048	−0.0050	−0.3140	0.1697	0.9460
X(15)	0.2919	−0.1037	−0.2823	1.2146	1.6509
X(16)	0.7190	0.0016	−0.2701	0.0075	0.5899
X(17)	0.8265	−0.0121	−0.5096	0.1204	0.9575
X(18)	0.9266	−0.0089	−0.2363	0.1172	0.9282
proper value	7.1660	1.4793	5.5731	1.7261	15.9444

4.8 Japan's export function towards the France
(Flexible Exchange-Rate System 1971III–1986IV)

<<Factor loadings by varimax rotation>> MFUL communality (Long term)

	1	2	3	4	communality
$X(1)$	–0.9775	–0.0961	–0.0064	0.1038	0.9755
$X(2)$	–0.9554	–0.0967	–0.0024	0.1614	0.9483
$X(3)$	–0.7435	–0.0784	0.0728	0.0399	0.5658
$X(4)$	0.9464	0.2704	–0.0095	0.0078	0.9689
$X(5)$	0.9523	0.2567	0.0117	–0.0510	0.9755
$X(6)$	0.9348	0.2545	0.0074	–0.1600	0.9643
$X(7)$	0.0249	–0.0298	–0.0691	–0.1829	0.0397
$X(8)$	0.4136	0.7398	0.0419	–0.1205	0.7347
$X(9)$	0.4042	0.7528	–0.0077	–0.0467	0.7323
$X(10)$	0.0048	–0.0515	0.6953	0.2196	0.5343
$X(11)$	–0.0532	0.0497	0.7532	–0.0799	0.5790
$X(12)$	–0.0072	–0.0215	0.6841	0.0793	0.4749
$X(13)$	0.8037	0.6009	0.0046	0.0086	1.0071
$X(14)$	0.9261	0.3917	–0.0038	–0.0167	1.0113
$X(15)$	0.6939	–0.2767	–0.2176	0.3605	0.7353
$X(16)$	0.2077	–0.4436	0.0513	–0.3244	0.3478
$X(17)$	0.2327	0.9028	–0.0145	0.1859	0.9039
proper value	7.5176	2.9518	1.5812	0.4479	12.4985

<<Factor loadings by varimax rotation>> MFUS communality (Short term)

	1	2	3	4	communality
$X(1)$	–0.9632	–0.1657	–0.0108	–0.1281	0.9717
$X(2)$	–0.9406	–0.1662	–0.0059	–0.1796	0.9447
$X(3)$	–0.7367	–0.1416	0.0723	–0.0328	0.5691
$X(4)$	0.9212	0.3347	–0.0039	0.0233	0.9612
$X(5)$	0.9266	0.3206	0.0170	0.0810	0.9683
$X(6)$	0.9095	0.3249	0.0100	0.1690	0.9614
$X(7)$	0.0246	–0.0216	–0.0706	0.1705	0.0351
$X(8)$	0.3548	0.7835	0.0421	0.1201	0.7559
$X(9)$	0.3485	0.7780	–0.0053	0.0539	0.7296
$X(10)$	0.0104	–0.0477	0.6921	–0.2192	0.5295
$X(11)$	–0.0626	0.0425	0.7574	0.0853	0.5867
$X(12)$	–0.0059	–0.0206	0.6824	–0.0856	0.4734
$X(13)$	0.7591	0.6559	0.0091	0.0112	1.0068
$X(14)$	0.8940	0.4573	0.0004	0.0344	1.0095
$X(15)$	0.7209	–0.2287	–0.2113	–0.3310	0.7262
$X(16)$	0.2349	–0.4411	0.0557	0.3759	0.3942
$X(17)$	0.1713	0.9029	–0.0106	–0.1722	0.8744
$X(18)$	0.4832	0.7802	–0.0299	–0.1277	0.8594
proper value	7.3549	3.9298	1.5798	0.4926	13.3571